ALLIGATION AND MIXTURE

Alligation means mixing two or more ingredients in some ratio. Problems based on alligation and mixture are asked in various competitive exams.

There are mainly two formulas used in alligation and mixture.

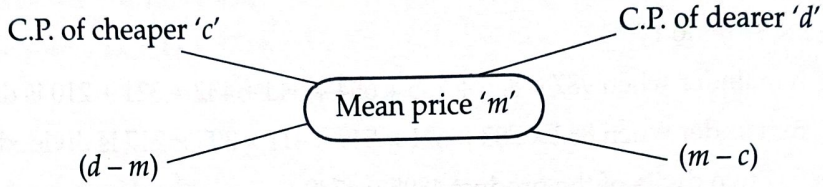
Formula-1

This formula of alligation and enables us to find the ratio in which two or more ingredients at the given prices must be mixed to produce a mixture at desired price.

Suppose we have two ingredients of different prices. The price per unit of cheaper ingredient is denoted by 'c' and the price per unit of dearer ingredient is denoted by 'd'. We mix these two ingredients in some ratio to produce a mixture at desired price (also called mean price, say 'm', then

$$\frac{\text{quantity of cheaper ingredient}}{\text{quantity of dearer ingredient}} = \frac{\text{C.P. of dearer} - \text{mean price}}{\text{mean price} - \text{C.P. of cheaper}}.$$

The above formula can be represented by the following diagram which is easier to understand:



Hence, $\dfrac{\textit{quantity of cheaper ingredient}}{\textit{quantity of dearer ingredient}} = \dfrac{d-m}{m-c}$.

ILLUSTRATIVE EXAMPLES

Example 1. *In what ratio must a shopkeeper mix two types of oranges worth ₹ 60 per kg and ₹ 90 per kg respectively so as to get a mixture at ₹ 80 per kg?*

Solution. Here,

the C.P. of cheaper oranges 'c' = ₹ 60 per kg,

the C.P. of dearer oranges 'd' = ₹ 90 per kg

and the mean price of the mixture 'm' = ₹ 80 per kg.

So, $\dfrac{\text{quantity of cheaper oranges}}{\text{quantity of dearer oranges}} = \dfrac{d-m}{m-c}$

$$= \frac{90-80}{80-60} = \frac{10}{20} = \frac{1}{2}.$$

Hence, the required ratio is 1 : 2.

Example 2. *The cost price of type I rice is ₹ 60 per kg and that of type II is ₹ 80 per kg. If both types of rice are mixed in the ratio 2 : 3 respectively, then find the price per kg of the mixed rice.*

Solution. Given, c = ₹ 60 per kg, d = ₹ 80 per kg

Quantity of cheaper : Quantity of dearer = 2 : 3

Let the price of mixed rice be ₹ m per kg.

$$\frac{\text{Quantity of cheaper rice}}{\text{Quantity of dearer rice}} = \frac{d-m}{m-c}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{80-m}{m-60}$$

$$\Rightarrow \quad 2m - 120 = 240 - 3m \Rightarrow 5m = 360$$

$\Rightarrow \quad m = \frac{360}{5} \Rightarrow m = 72.$

Hence, the price of mixed rice is ₹ 72 per kg.

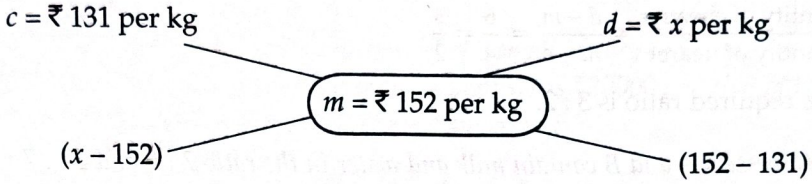
Example 3. *Three varieties A, B and C of tea are mixed together in the ratio 1 : 1 : 3 respectively. The price of tea A is ₹ 127 per kg and that of tea B is ₹ 135 per kg. If the price of the mixture is ₹ 152 per kg, then find the price per kg of tea C.*

Solution. Since tea A and tea B are mixed in equal proportion, so their average price is $\frac{127 + 135}{2}$ *i.e.* ₹ 131 per kg.

As the price per kg of the mixture is ₹ 152 per kg which is more than the average price of tea A and tea B. So the price per kg of tea C must be greater than ₹ 152. Let the price of tea C be ₹ x per kg.

(Tea A + tea B) and tea C are mixed in the ratio (1 + 1) : 3 *i.e.* 2 : 3.

Here,



$\therefore \quad \frac{2}{3} = \frac{x - 152}{152 - 131} \Rightarrow 3x - 456 = 42$

$\Rightarrow \quad 3x = 498 \Rightarrow x = 166.$

Hence, the price of tea C is ₹ 166 per kg.

Example 4. *A vessel contains a mixture of two liquids P and Q in the ratio 5 : 7. 12 litres of mixture are drawn off from the vessel and 12 litres of liquid P is filled in the vessel. If the ratio of liquids P and Q now becomes 9 : 7, how many litres of liquids P and Q were contained by the vessel initially?*

Solution. Let initially liquids P and Q be $5x$ litres and $7x$ litres respectively in the vessel.

After drawing off 12 litres of mixture

Quantity of liquid P left in the mixture $= 5x - \frac{5}{12} \times 12$

$= (5x - 5)$ litres

Quantity of liquid Q left in the mixture $= 7x - \frac{7}{12} \times 12$

$= (7x - 7)$ litres.

Further, 12 litres of liquid P are mixed in the mixture.

So, quantity of liquid P = $(5x - 5 + 12)$ litres

$= (5x + 7)$ litres

and quantity of liquid Q = $(7x - 7)$ litres

According to given, $\frac{9}{7} = \frac{5x + 7}{7x - 7}$

$\Rightarrow \quad 63x - 63 = 35x + 49$

$\Rightarrow \quad 63x - 35x = 63 + 49$

$\Rightarrow \quad 28x = 112 \Rightarrow x = 4.$

Hence, the quantity of liquid P was 5×4 *i.e.* 20 litres and quantity of liquid Q was 7×4 *i.e.* 28 litres initially.

Example 5. *In what ratio must a grocer mix two varieties of sugar worth ₹ 40 per kg and ₹ 50 per kg respectively so that by the selling the mixture at ₹ 48.40 per kg, he may gain 10%?*

Solution. Let the C.P. of the mixture be ₹ x per kg.

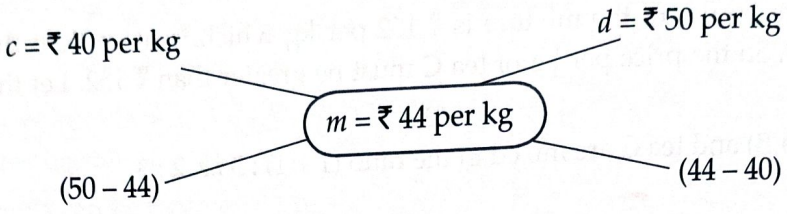
Then S.P. = C.P. + profit

$\Rightarrow$ $48.40 = x + 10\%$ of x

$\Rightarrow$ $48.40 = x + \frac{10x}{100} \Rightarrow 48.40 = \frac{11x}{10}$

$\Rightarrow$ $x = \frac{484}{11} \Rightarrow x = 44$

Now,



$\therefore$ $\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{d-m}{m-c} = \frac{6}{4} = \frac{3}{2}$.

Hence, the required ratio is 3 : 2.

Example 6. *Two vessels A and B contain milk and water in the ratio 7 : 5 and 17 : 7 respectively. In what ratio mixtures from two vessels should be mixed to get a new mixture containing milk and water in the ratio 5 : 3?*

Solution. Quantity of milk in mixture A = $\frac{7}{12}$ part,

quantity of milk in mixture B = $\frac{17}{24}$ part

and quantity of milk in new mixture = $\frac{5}{8}$ part

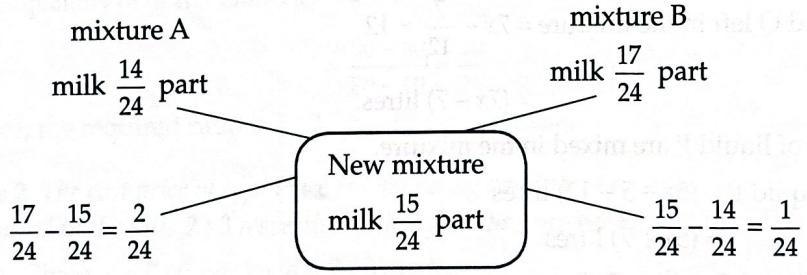
$\therefore$ L.C.M. of 12, 24 and 8 = 24.

$\therefore$ Quantity of milk in mixture A = $\frac{7\times2}{12\times2} = \frac{14}{24}$ part.

Quantity of milk in mixture B = $\frac{17}{24}$ part

Quantity of milk in new mixture = $\frac{5\times3}{8\times3} = \frac{15}{24}$ part

So,



$\therefore$ $\frac{\text{Quantity of mixture A}}{\text{Quantity of mixture B}} = \frac{\frac{2}{24}}{\frac{1}{24}} = \frac{2}{1}$.

Hence, the required ratio is 2 : 1.

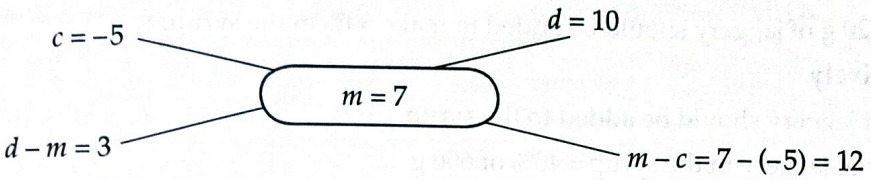
Example 7. *A retailer has 250 kg of rice a part of which he sells at 10% profit. The remaining quantity of rice is of low quality and he sold it at 5% loss. Overall, he made a profit of 7%. Find the quantity of rice sold at 5% loss.*

Solution. Given, total quantity of rice = 250 kg

$d = 10\%$ (profit)

$c = -5\%$ (loss)

$m = 7\%$ (overall profit)



$$\therefore \quad \frac{\text{Quantity of rice sold at 5\% loss}}{\text{Quantity of rice sold at 10\% profit}} = \frac{3}{12} = \frac{1}{4} \text{ i.e. } 1:4.$$

So, quantity of rice sold at 5% loss = $\frac{1}{5} \times 250$ kg = 50 kg.

Hence, the quantity of rice sold at 5% loss = 50 kg.

Example 8. *The average salary per head of the entire staff of a small factory including the supervisors and labours is ₹5750. The average salary per head of the supervisor is ₹20000 and that of labours is ₹5000. Find the number of labours in the factory if there are 4 supervisors.*

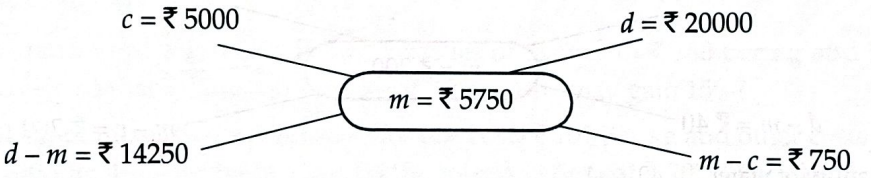
Solution. Given, average salary of labours (c) = ₹5000,

average salary of supervisor (d) = ₹20000

and average salary of the entire staff m = ₹5750.

Also number of supervisors = 4.

Let the number of labours be x.



$$\therefore \quad \frac{\text{Number of labours}}{\text{Number of supervisors}} = \frac{14250}{750}$$

$$\Rightarrow \quad \frac{x}{4} = \frac{14250}{750} \Rightarrow x = 19 \times 4 = 76.$$

Hence, the number of labours = 76.

Example 9. *600 g of jaggery syrup has 40% jaggery in it. How much jaggery should be added to make it 50% in the syrup?*

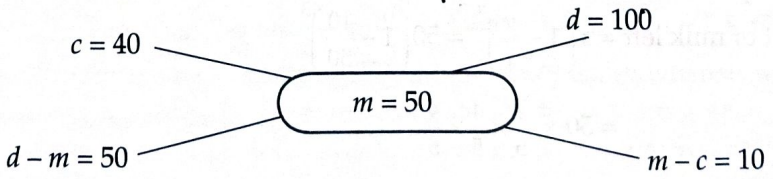
Solution. Given, jaggery in the syrup (c) = 40%

Pure jaggery (d) = 100%

Resulting jaggery in the syrup (m) = 50%

Quantity of jaggery syrup = 600 g

Let the quantity of jaggery be x g, then



$$\therefore \quad \frac{\text{Quantity of jaggery syrup}}{\text{Quantity of pure jaggery}} = \frac{50}{10}$$

$$\Rightarrow \quad \frac{600}{x} = \frac{5}{1} \Rightarrow x = 120 \text{ g}.$$

Hence, 120 g of jaggery should be added to make 50% in the syrup.

Alternatively

Let x g of jaggery should be added to the syrup.

Quantity of jaggery in the syrup = 40% of 600 g

$$= \frac{40}{100} \times 600 \text{ g} = 240 \text{ g}.$$

So, $240 + x = (600 + x) \times \frac{50}{100}$

$$\Rightarrow \quad 240 + x = 300 + \frac{x}{2} \Rightarrow x - \frac{x}{2} = 300 - 240$$

$$\Rightarrow \quad \frac{x}{2} = 60 \Rightarrow x = 120.$$

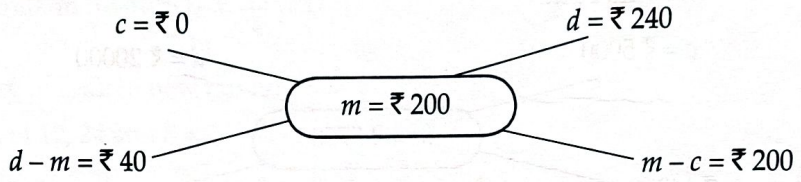
Hence, 120 g of jaggery should be added.

Example 10. *In what ratio water must be added to dilute honey costing ₹240 per litre so that the resulted syrup would be worth ₹200 per litre?*

Solution. Cost of water (c) = ₹0 per litre

Cost of honey (d) = ₹240 per litre

Cost of honey syrup (m) = ₹200 per litre



$$\therefore \quad \frac{\text{Quantity of water}}{\text{Quantity of honey}} = \frac{40}{200} = \frac{1}{5}.$$

Hence, the required ratio is 1 : 5.

Formula-2 (Repeated dilution)

This formula is used to calculate the pure quantity of a liquid left after 'n' number of repeated dilution.

Suppose a vessel contains 'x' units of a liquid from which y units are taken out and replaced by water. ***After 'n' number of repeated dilutions the quantity of pure liquid*** $= x\left(1 - \frac{y}{x}\right)^n$ ***units.***

Example 11. *A container contains 50 litres of milk. From this container 10 litres of milk was taken out and replaced by water. This process is repeated two more times. How much milk is now left in the container?*

Solution. Given $x = 50$ litres, $y = 10$ litres, $n = 3$.

So, amount of milk left $= x\left(1 - \frac{y}{x}\right)^n = 50\left(1 - \frac{10}{50}\right)^3$

$$= 50 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$$

$$= \frac{128}{5} = 25.6 \text{ litres}.$$

Example 12. *From a cask full of alcohol 8 litres are drawn and then filled with water. This process is repeated three more times. The ratio of the quantity of alcohol left in the cask and that of water is 16 : 65. How many litres of alcohol did the cask hold originally?*

Solution. Let the quantity of alcohol in the cask be x litres originally.

So, the quantity of alcohol left in the cask after 4 processes of dilution $= x\left(1-\frac{8}{x}\right)^4$ litres

$$\therefore \quad \frac{x\left(1-\frac{8}{x}\right)^4}{x} = \frac{16}{81}$$

$$\Rightarrow \quad \left(1-\frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^4$$

$$\Rightarrow \quad 1-\frac{8}{x} = \frac{2}{3} \Rightarrow \frac{8}{x} = \frac{1}{3}$$

$$\Rightarrow \quad x = 24$$

Hence, the quantity of alcohol in the cask was 24 litres originally.

EXERCISE 1.2

1. In what ratio must a grocer mix two varieties of pulses worth ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth ₹ 92 per kg?
2. The cost price of type A apples is ₹ 120 per kg and that of type B apples is ₹ 180 per kg. If both types of apples are mixed in the ratio 2 : 3 respectively, then find the price per kg of mixed apples.
3. In what ratio must rice at ₹ 45 per kg is mixed with rice at ₹ 60 per kg so that the mixture be worth ₹ 54 per kg?
4. In what ratio must a grocer mix two varieties of tea worth ₹ 180 per kg and ₹ 200 per kg respectively so that selling the mixture at ₹ 216.20, he may gain 15%?
5. A shopkeeper mixes two varieties of rice one costs ₹ 161 per kg and other costs ₹ 179 per kg in the ratio 5 : 4 respectively. He sells the mixed variety at ₹ 202.80 per kg. Find his profit percentage.
6. How many kg of sugar costing ₹ 45 per kg must be mixed with 30 kg sugar costing ₹ 35 per kg so that there may be a gain of 12% by selling the mixture at ₹ 47.04 per kg?
7. Three varieties A, B and C of rice are mixed together in the ratio 4 : 1 : 1 respectively. The cost price of rice B is ₹ 82 per kg and that of rice C is ₹ 90 per kg. If the price of the mixture is ₹ 94 per kg, then find the price per kg of rice A.
8. A vessel contains a mixture of two liquids X and Y in the ratio 3 : 5. 8 litres of mixture are drawn off from the vessel and 8 litres of liquid X is filled in the vessel. If the ratio of liquids X and Y is now becomes 7 : 10, how many litres of liquids X and Y were contained by the vessel initially?
9. Two vessels P and Q contain milk and water in the ratio 5 : 3 and 13 : 3 respectively. In what ratio mixtures from two vessels should be mixed to get a new mixture containing milk and water in the ratio 3 : 1 respectively?
10. Two vessels A and B contain milk and water in the ratio 3 : 2 and 7 : 13 respectively. They are mixed in the ratio 2 : 3. Find the ratio of milk and water in the final mixture.
11. A milkman has two cans. First containing 75% milk and rest water, whereas second containing 50% milk and rest water. How much mixture should he mix from each can so as to get 20 litres of mixture, such that ratio of milk and water is 5 : 3 respectively?
12. In what ratio must a person mix two sugar solutions of 30% and 50% concentration respectively so as to get a solution of 45% concentration?

13. A shopkeeper has 1 quintal of wheat, part of which she sells at 18% gain and the rest at 28% gain. In total she gains 24%. Find the quantity of wheat sold at 18% and 28%.
14. The average salary per worker of the entire staff of a polythene bag manufacturing unit including machine operator and labours is ₹7500. The average salary per head of the machine operator is ₹25000 and that of labours is ₹6000. Find the number of workers in the unit if there are 3 machine operators.
15. 900 g of sugar solution has 40% sugar in it. How much sugar should be added to make it 55% in the solution?
16. In what ratio water must be added in milk costing ₹60 per litre, so that resulting mixture would be worth ₹50 per litre?
17. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated two more times. How much milk is now left in the container?
18. From a container containing 50 litres of milk, 8 litres of milk was taken out and replaced by water. This process is repeated two more times. How much milk is now left in the container?
19. From a container full of alcohol 6 litres of alcohol was drawn out and replaced by water. This process is repeated one more time. The ratio of the quantity of alcohol and water left in the container is 9 : 16. How many litres of alcohol did the container hold originally?

Answers

1. 8 : 7 **2.** ₹ 156 **3.** 2 : 3 **4.** 3 : 2 **5.** 20% **6.** 70 kg
7. ₹ 98 **8.** Liquid X = 51 litres, liquid Y = 85 litres **9.** 1 : 2 **10.** 9 : 11
11. 10 litres, 10 litres **12.** 1 : 3 **13.** 40 kg, 60 kg **14.** 38
15. 300 g **16.** 1 : 5 **17.** 29.16 litres **18.** 29.635 litres **19.** 15 litres